Extension of scalar-tensor theories

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2019 CCNU-USTC Junior Cosmology Symposium

Why modified gravity

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Phenomenological:

To explain the early and late accelerated expansion of our universe.

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Theoretical:

To understand why GR is unique.

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To explain the early and late accelerated expansion of our universe.

Theoretical:

To understand why GR is unique.

"The best way to understand something is to modify it."

How to modify gravity

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Einstein equation is quite unique.

$$\alpha G_{\mu\nu} + \lambda g_{\mu\nu} = 0$$

[Lovelock, 1971]

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Any metric theory of gravity alternative to GR must satisfy (at least):

- extra degrees of freedom,
- extra dimensions (e.g., brane world),
- higher derivative terms (e.g., $f(R)$),
- extension of Riemannian geometry (e.g., $f(T)$),
- giving up locality.

How to modify gravity

Einstein equation is quite unique.

$$\alpha G_{\mu\nu} + \lambda g_{\mu\nu} = 0$$

[Lovelock, 1971]

Any metric theory of gravity alternative to GR must satisfy (at least):

- extra degrees of freedom (with exotic couplings with gravity),
- extra dimensions (e.g., brane world),
- higher derivative terms (e.g., $f(R)$),
- extension of Riemannian geometry (e.g., $f(T)$),
- giving up locality.

Covariant scalar-tensor theories

From *k*-essence to Horndeski and beyond

1915 • GR

$$\mathcal{L} = \frac{1}{16\pi G} R$$

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1961 Brans-Dicke

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$$\mathcal{L} = \phi R - \frac{\omega}{\phi} (\partial\phi)^2$$

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[Chiba, Okabe, Yamaguchi, 1999]

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$$\mathcal{L} = g\left(\phi\right) R + F\left(\phi, \partial\phi\right)$$

From *k*-essence to Horndeski and beyond

1915	GR	$\mathcal{L} = \frac{1}{16\pi G} R$
1961	Brans-Dicke *[Brans & Dicke, 1961]*	$\mathcal{L} = \phi R - \frac{\omega}{\phi} (\partial\phi)^2$
1999	*k*-essence *[Chiba, Okabe, Yamaguchi, 1999]* *[Armendariz-Picon, Damour, Mukhanov, 1999]*	$\mathcal{L} = g(\phi) R + F(\phi, \partial\phi)$

k-essence:
the most general scalar-tensor theory,
the Lagrangian involves up to the first derivative of the scalar field.

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Beyond *k*-essence?

Introducing second derivatives?

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→ Ostrogradsky's ghost

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[Deffayet, XG, Steer & Zahariade

***Phys.Rev. D** 84 (2011) 064039]*

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$$\begin{aligned}\mathcal{L} = {} & G_2(X,\phi) + G_3(X,\phi)\Box\phi \\ & + G_4(X,\phi)\, R + \frac{\partial G_4}{\partial X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\ & + G_5(X,\phi)\, G^{\mu\nu}\nabla_\mu\nabla_\nu\phi \\ & - \frac{1}{6}\frac{\partial G_5}{\partial X}\left[(\Box\phi)^3 - 3\Box\phi\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right]\end{aligned}$$

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$$\nabla_\nu\phi)^3\big]$$

Generalized galileon/Horndeski theory: the most general scalar-tensor theory,

- Lagrangian/EoMs involve up to the second derivatives,
- Propagates 1 scalar + 2 tensor dofs.

From *k*-essence to Horndeski

From k-essence to generalised Galileons

C. Deffayet (APC, Paris & Paris U., VI-VII), Xian Gao (APC, Paris & Paris U., VI-VII & Ecole Normale Superieure & Paris, Inst. Astrophys.), D.A. Steer, G. Zahariade (APC, Paris & Paris U., VI-VII). Mar 2011. 25 pp.

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e-Print: **arXiv:1103.3260 [hep-th]** | **PDF**

References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service

レコードの詳細 - Cited by 630 records 500+

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International Journal of Theoretical Physics, Vol. 10, No. 6 (1974), pp. 363–384

Second-Order Scalar-Tensor Field Equations in a Four-Dimensional Space

GREGORY WALTER HORNDESKI

Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario, Canada

Received: 10 *July* 1973

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Beyond galileon/Horndeski theory?

- Higher derivatives of the scalar field and the metric.
- Propagates 1 scalar + 2 tensor dofs.

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2015 DHOST

[Langlois & Noui, 2015]

[Crisostomi, Koyama & Tasinato, 2016]

DHOST (degenerate higher order scalar-tensor) theory:

- Higher derivatives in EoMs (2nd derivatives in the action),
- Propagates 1 scalar + 2 tensor dofs.

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Even beyond?

In *D*=4：

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2015 DHOST

[Langlois & Noui, 2015]

We may need some alternative approach.

Spatially covariant gravity

Spatially covariant gravity

“不忘初心，方得始终。”

“Never forget why you started,
and your mission can be accomplished.”

「初心忘るべからず、終始心得るべし。」

Spatially covariant gravity

初心 (beginner's mind) = a scalar degree of freedom

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Reduction of symmetries → Extra mode(s)

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Breaking U(1) → Massive vector (2 vector + 1 scalar)

Spatially covariant gravity

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Reduction of symmetries → Extra mode(s)

Breaking U(1) → Massive vector (2 vector + 1 scalar)

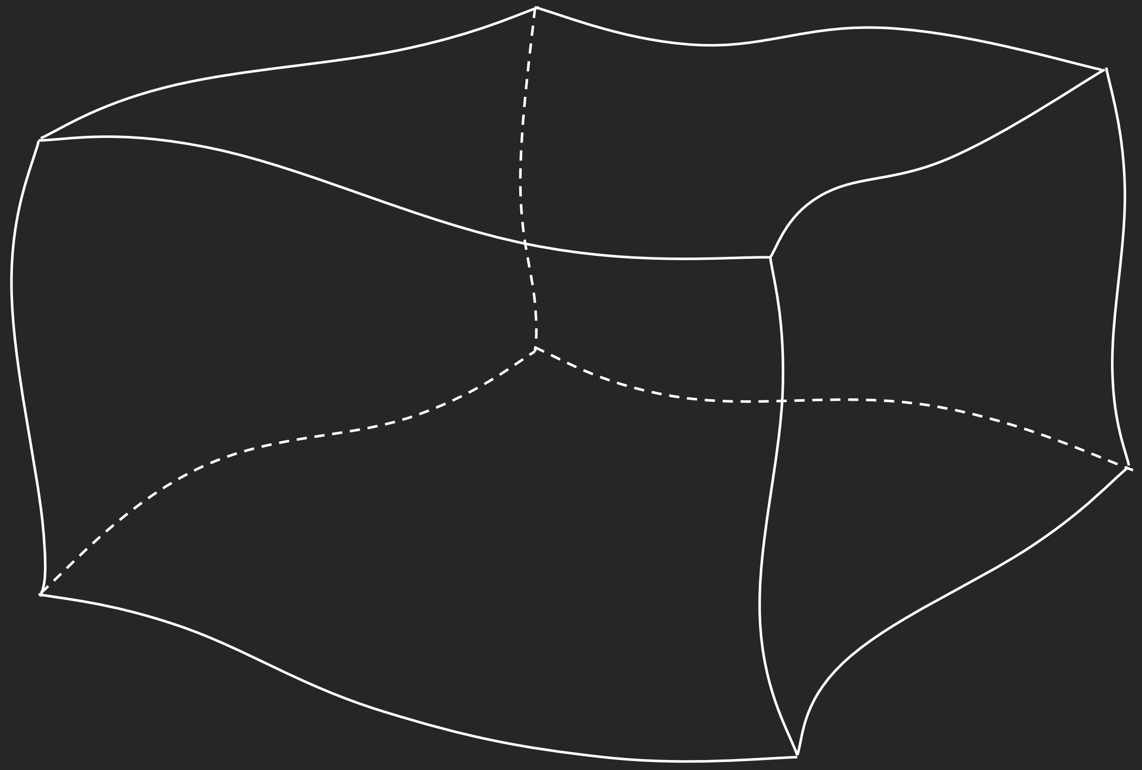
Breaking whole spacetime diff → Massive gravity (2 tensor + 2 vector + 1 scalar)

Spatially covariant gravity

初心 (beginner's mind) = a scalar degree of freedom

Reduction of symmetries	→	Extra mode(s)
Breaking U(1)	→	Massive vector (2 vector + 1 scalar)
Breaking whole spacetime diff	→	Massive gravity (2 tensor + 2 vector + 1 scalar)
Breaking time diff, respecting only spatial symmetries	→	2 tensor + 1 scalar

Foliation of spacetime



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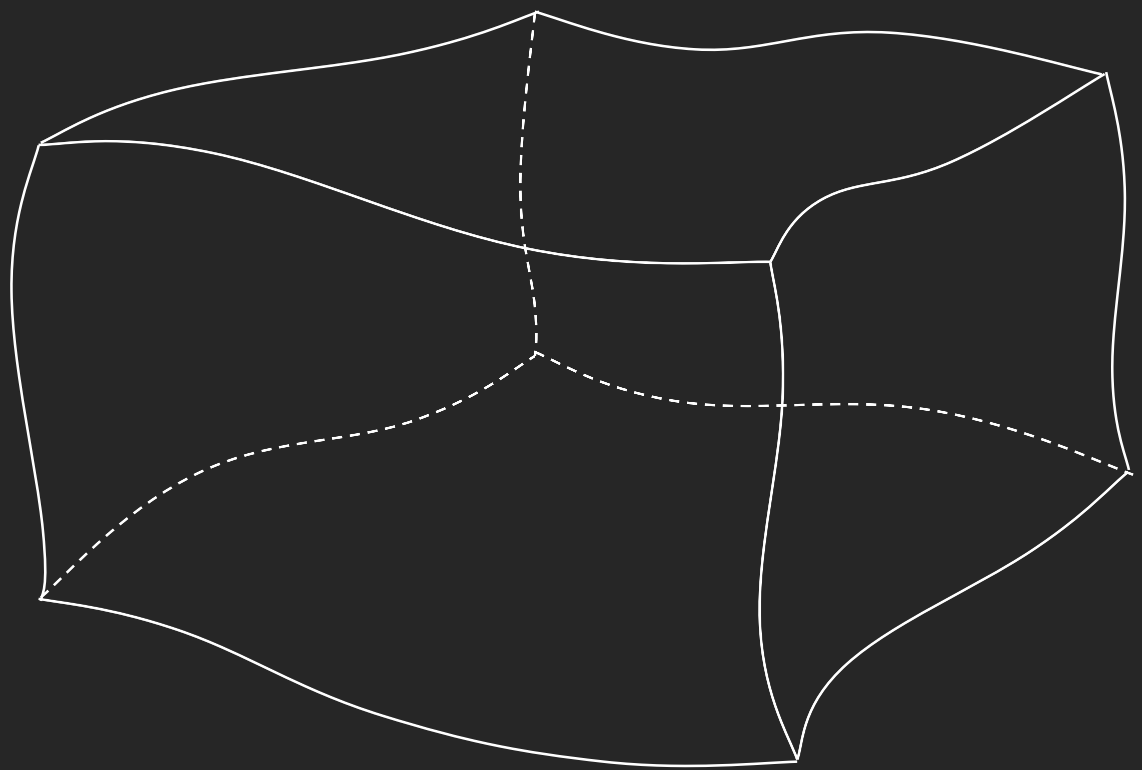
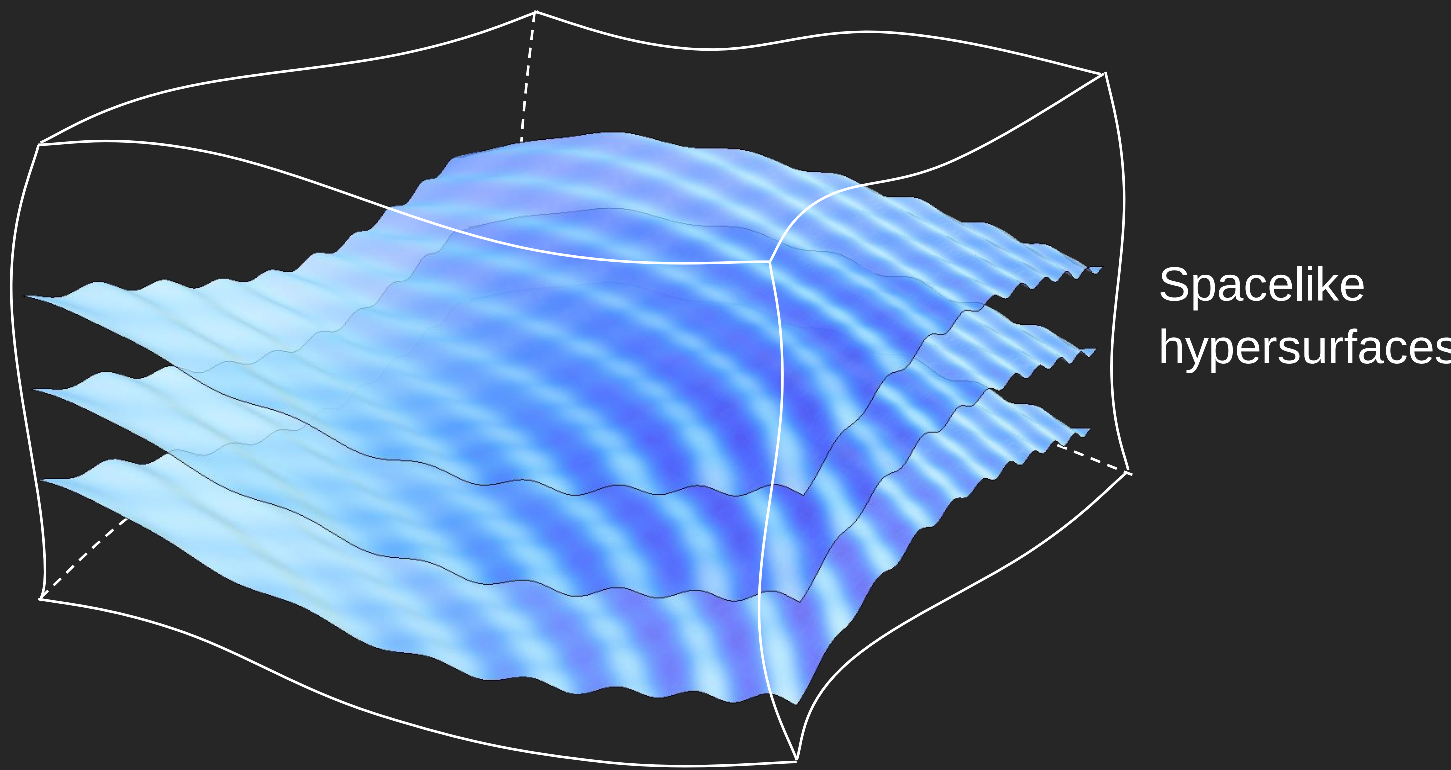


Spacetime covariant

4-D quantities

$\phi, g_{\mu\nu}, R_{\mu\nu\rho\sigma}, \nabla_\mu$

Foliation of spacetime



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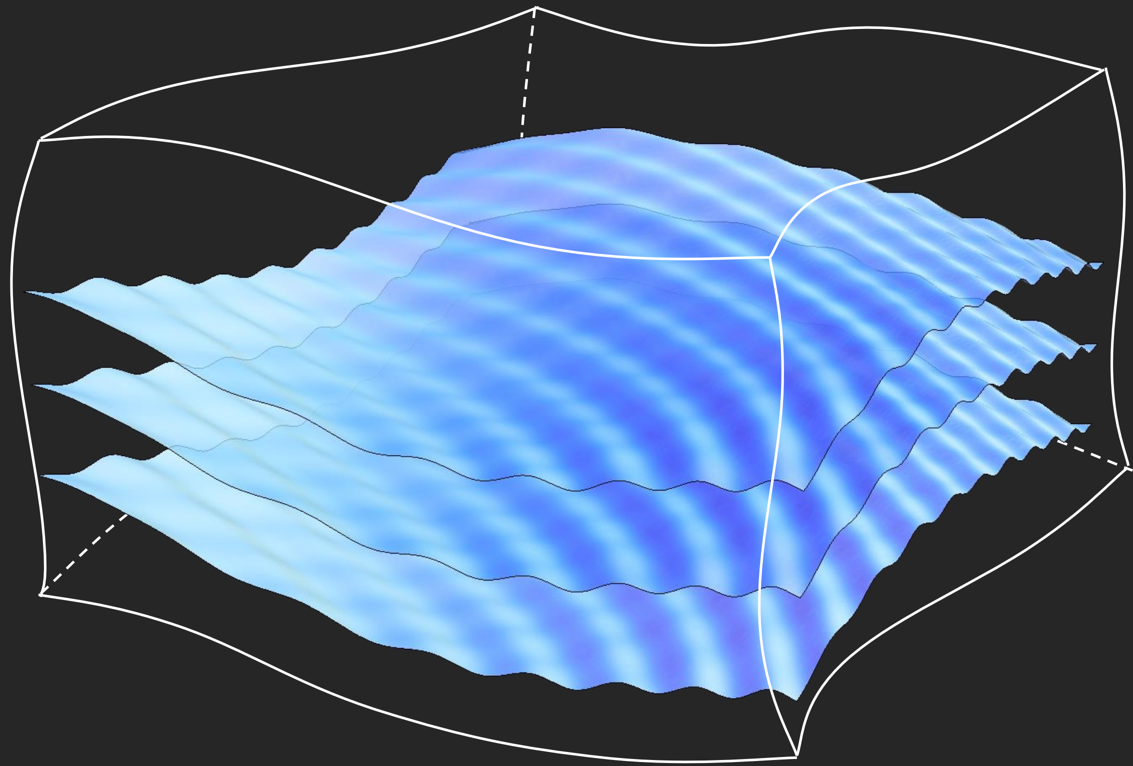

Spacetime covariant

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➡

Spatially covariant

3-D quantities

$t, N, h_{ij}, R_{ij}, \nabla_i, K_{ij}$

Early examples

2004 Ghost condensation

[Arkani-Hamed, Cheng, Luty & Mukohyama]

2007 Effective field theory of inflation

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- The Lagrangians are built of spatial invariants;
- The theories propagates one scalar mode (besides the two tensor modes).

Two faces of scalar-tensor theories

Spacetime covariant
Scalar-tensor theories

$$\mathcal{L}(\phi, g_{\mu\nu}, R_{\mu\nu\rho\sigma}, \nabla_\mu)$$

Two faces of scalar-tensor theories

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Spatially covariant
gravity theories

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Two faces of scalar-tensor theories

Gauge fixing: $\phi(t, \vec{x}) \to t$
(unitary gauge)

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Spacetime covariant Scalar-tensor theories

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Spatially covariant gravity theories

$$\mathcal{L}(t, N, h_{ij}, R_{ij}, K_{ij}, \nabla_i)$$

Gauge recovering: $t \to \phi(t, \vec{x})$
(Stueckelberg trick)

[H. Motohashi, T. Suyama, K. Takahashi, 2016]
[A. De Felice, D. Langlois, S. Mukohyama, K. Noui & A. Wang, 2018]

Beyond Horndeski

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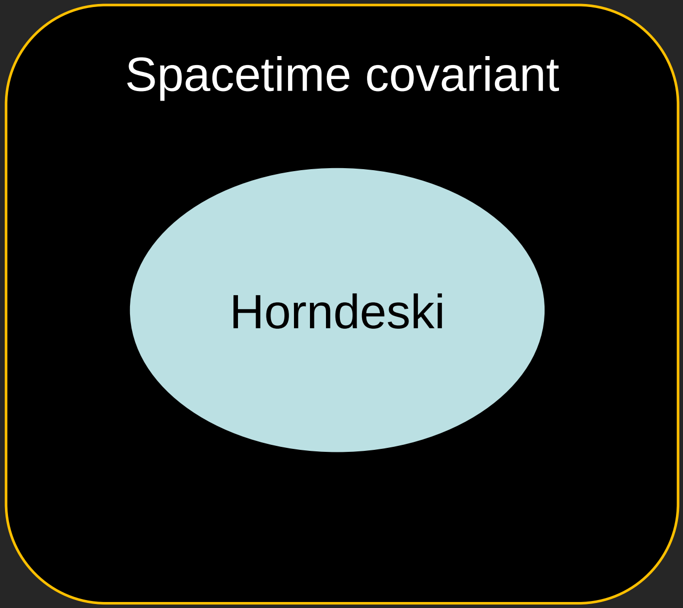

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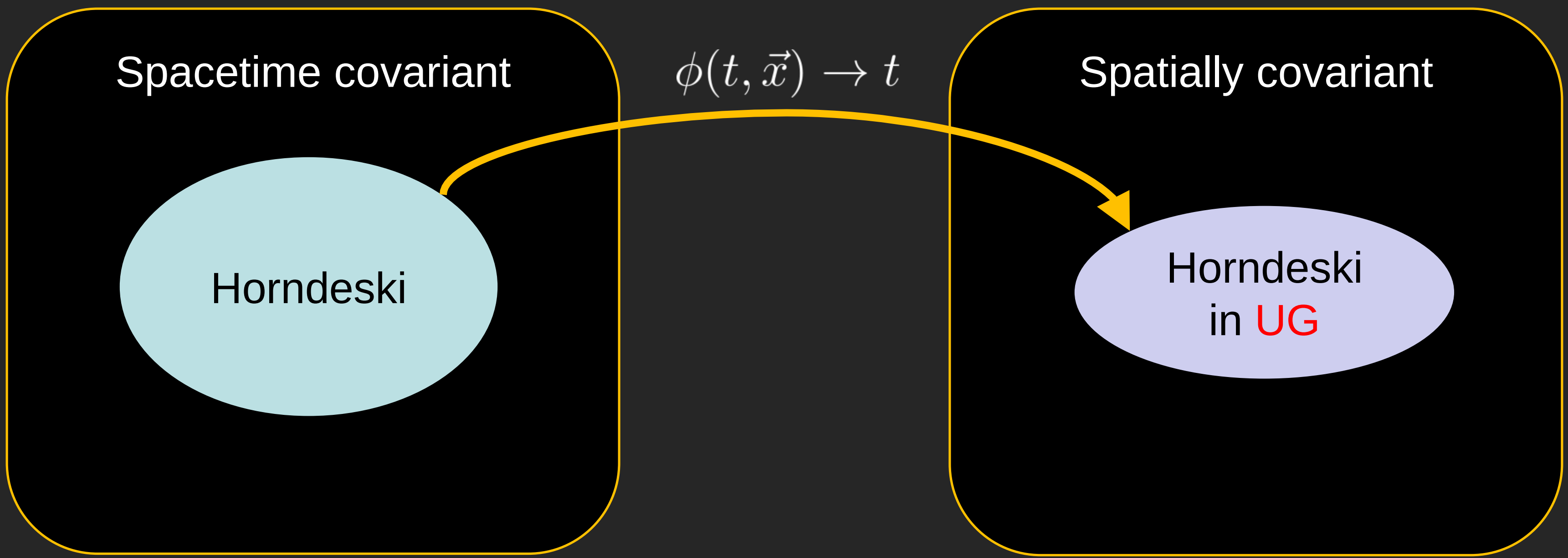

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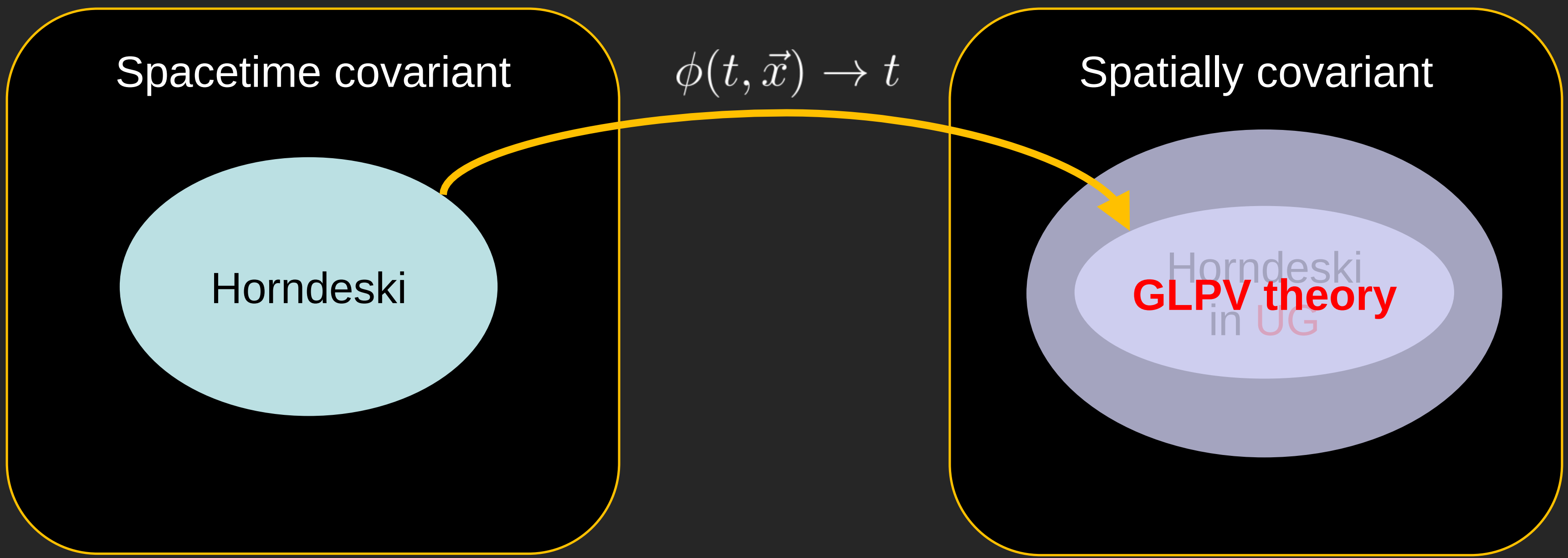

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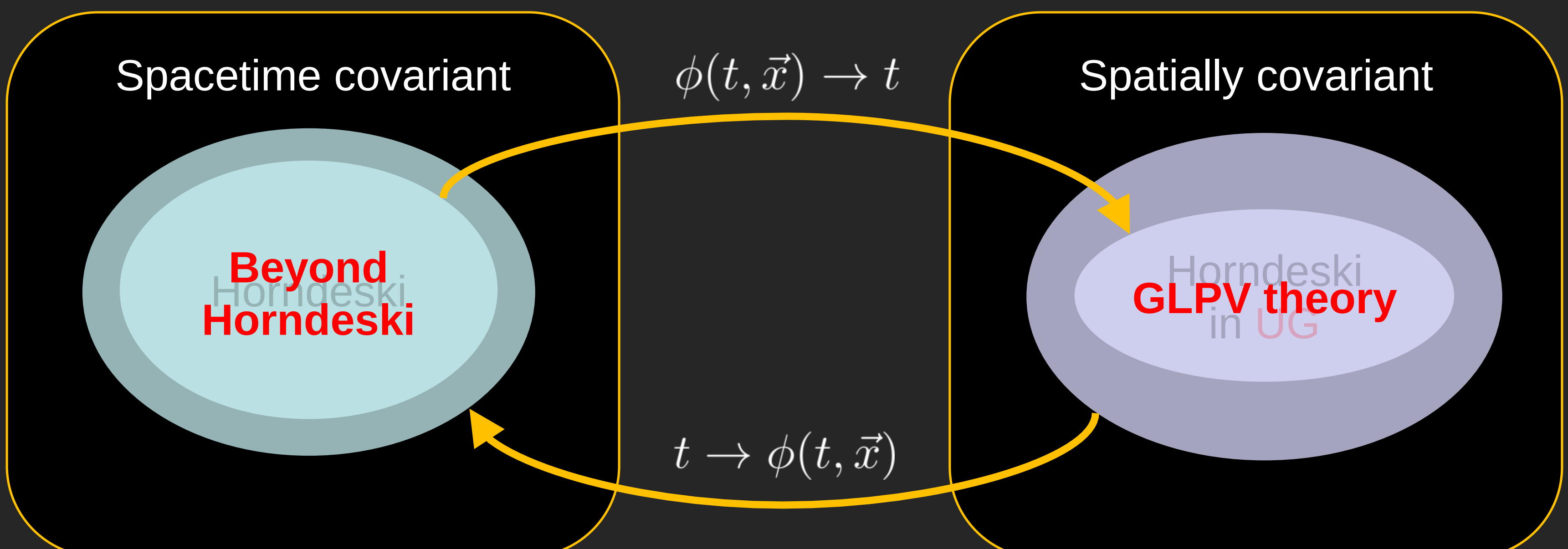

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*[XG, **Phys.Rev. D** 90 (2014) 081501 (**Rapid Communication**)]*

$$S=\int \mathrm{d}t\mathrm{d}^3x\,N\sqrt{h}\mathcal{L}(t,N,h_{ij},R_{ij},K_{ij},\nabla_i)$$

Spatially covariant gravity

2004 **Ghost condensation**
[Arkani-Hamed, Cheng, Luty & Mukohyama]

2007 **Effective field theory of inflation**
[Cheung, Creminelli, Fitzpatrick, Kaplan & Senatore]

2009 **Hořava gravity**
[Horava]

2014 **GLPV theory**
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2 tensor + 1 scalar DoFs with higher derivative EoMs.
[XG, Phys.Rev. D90 (2014) 104033]

With Velocity of the lapse function

Geometric motivation

The basic picture:

4d spacetime + foliation of spacelike hypersurfaces

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$\phi,\ N,\ h_{\mu\nu}$ with derivatives in terms of

- $\pounds_{\boldsymbol{n}}$ time der.
- D_μ space der.

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Also, field transformations typically introduces $\dot{N}$.

[G. Domènech, S. Mukohyama, R. Namba, A. Naruko, R. Saitou, Y. Watanabe, 2015]

The action

[XG & Zhi-Bang Yao, arXiv:1806.02811]

General action (in the unitary gauge):

$$S=\int \mathrm{d}t\mathrm{d}^3x\, N\sqrt{h}\,\mathcal{L}\left(t,N,h_{ij},F,K_{ij},\nabla_i\right)$$

with $$F=\frac{1}{N}\left(\dot{N}-\pounds_{\vec{N}}N\right),\qquad K_{ij}=\frac{1}{2N}\left(\dot{h}_{ij}-\pounds_{\vec{N}}h_{ij}\right)$$

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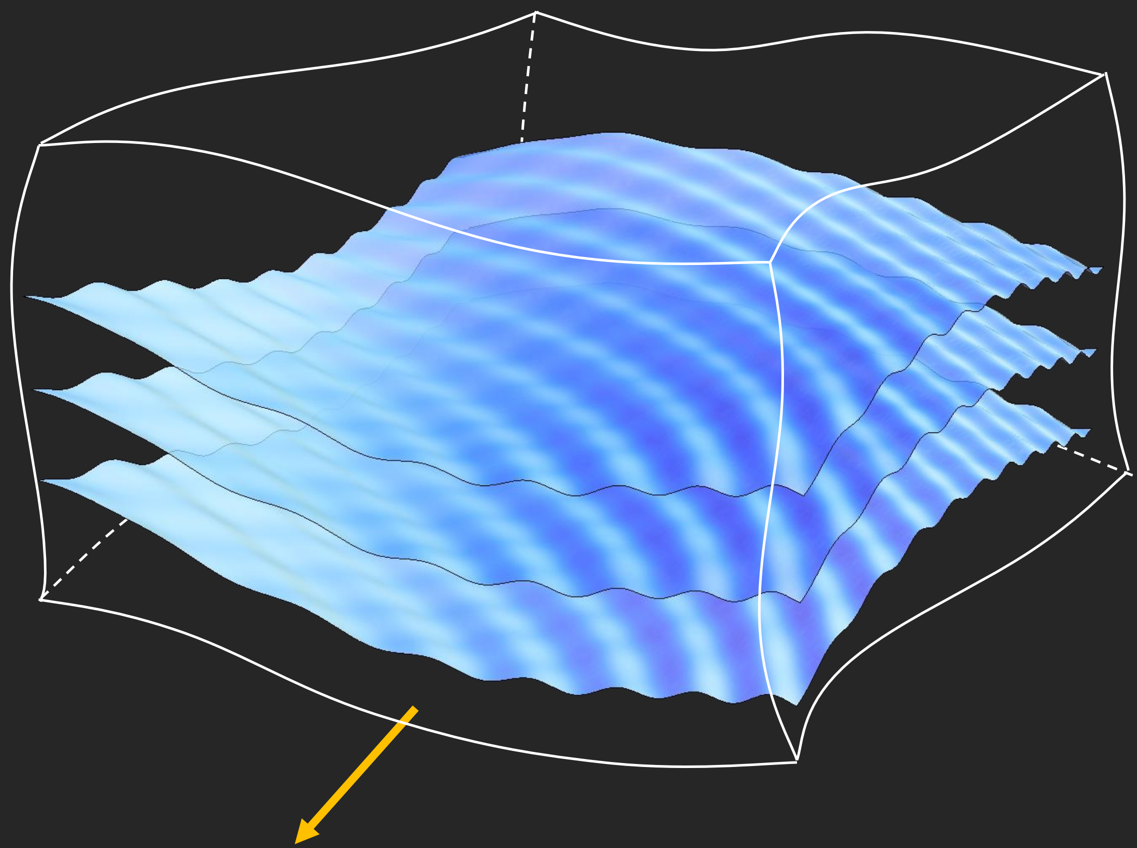
Nevertheless, we found conditions to kill the ghost.

[*See Zhi-bang Yao's talk*]

With a non-dynamical scalar field

How if spacelike?

How if the scalar field acquires a spacelike gradient?



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spacelike hypersurfaces

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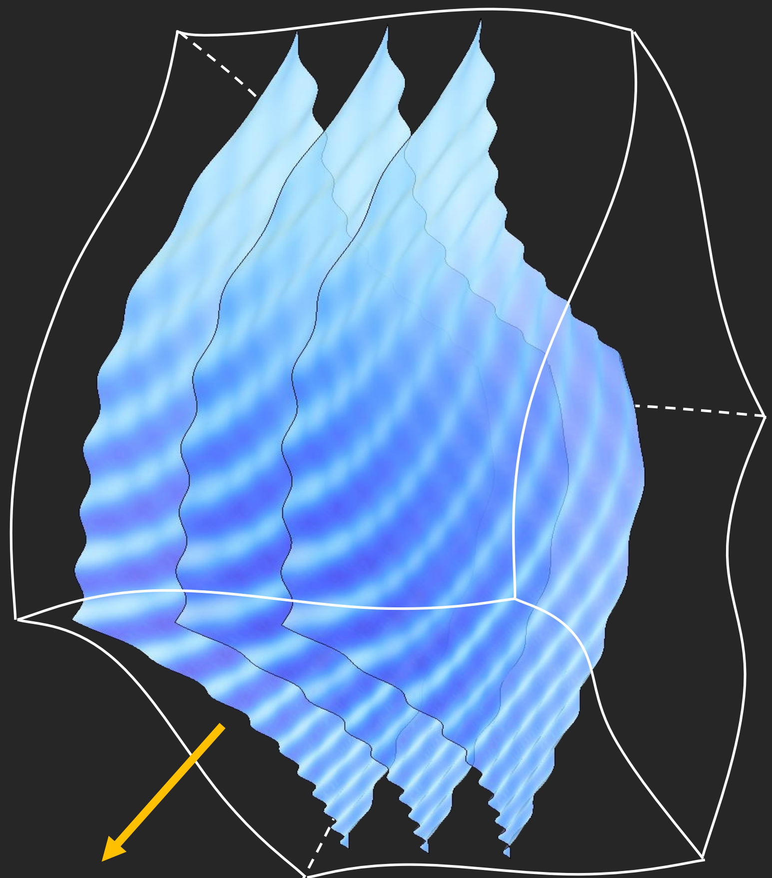

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timelike hypersurfaces

Spatial gauge

timelike spacelike

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Spatial gauge

timelike | spacelike

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unitary gauge

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*[**XG**, M. Yamaguchi, D. Yoshida, JCAP 1903 (2019) 006]*

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- ϕ is a non-dynamical (auxiliary) field

Spatial gauge

timelike — spacelike

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- ϕ is a non-dynamical (auxiliary) field
- General covariance is broken, which leads to a scalar d.o.f.

Evolution of the theories

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1915 GR

1961 Brans-Dicke

1999 *k*-essence

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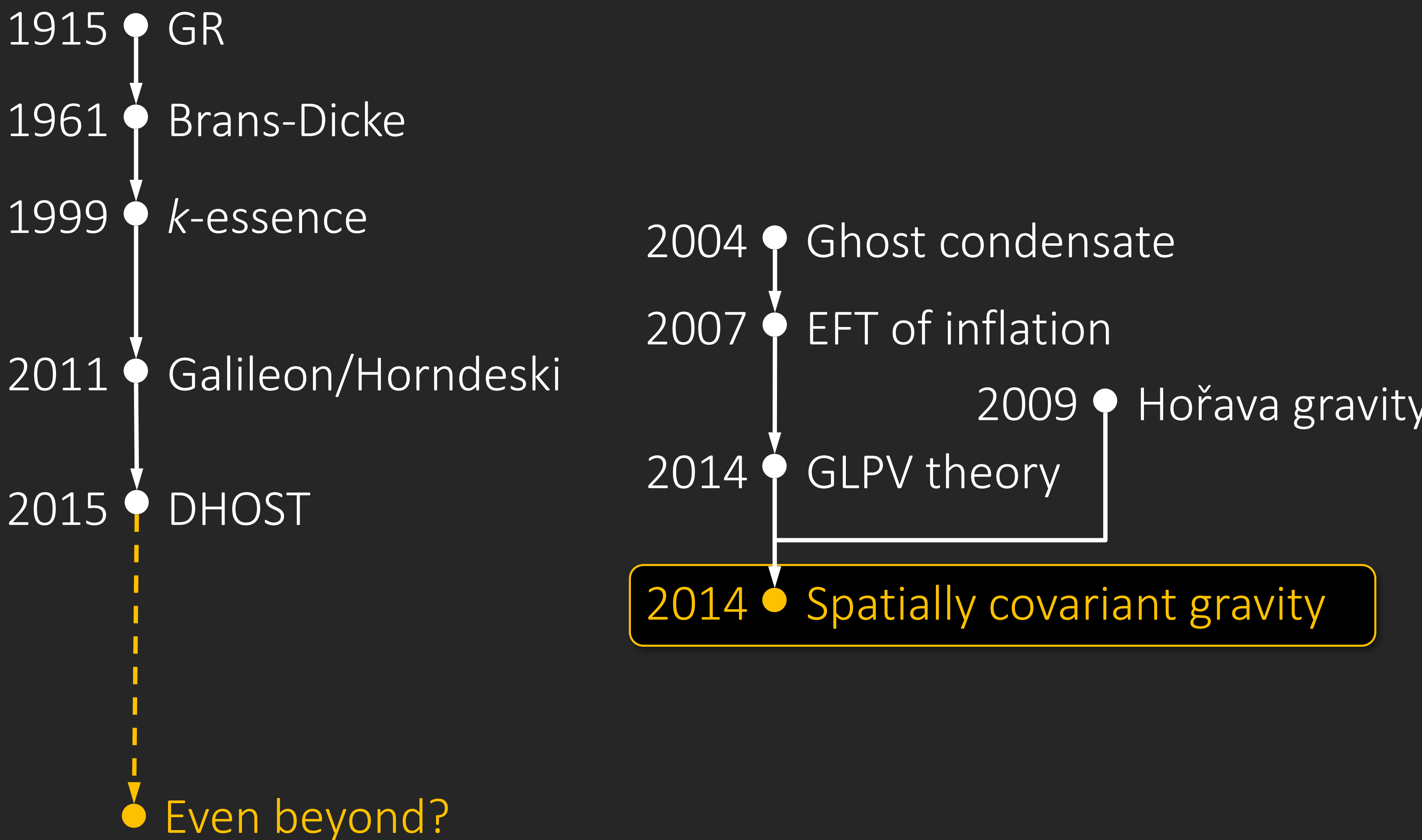

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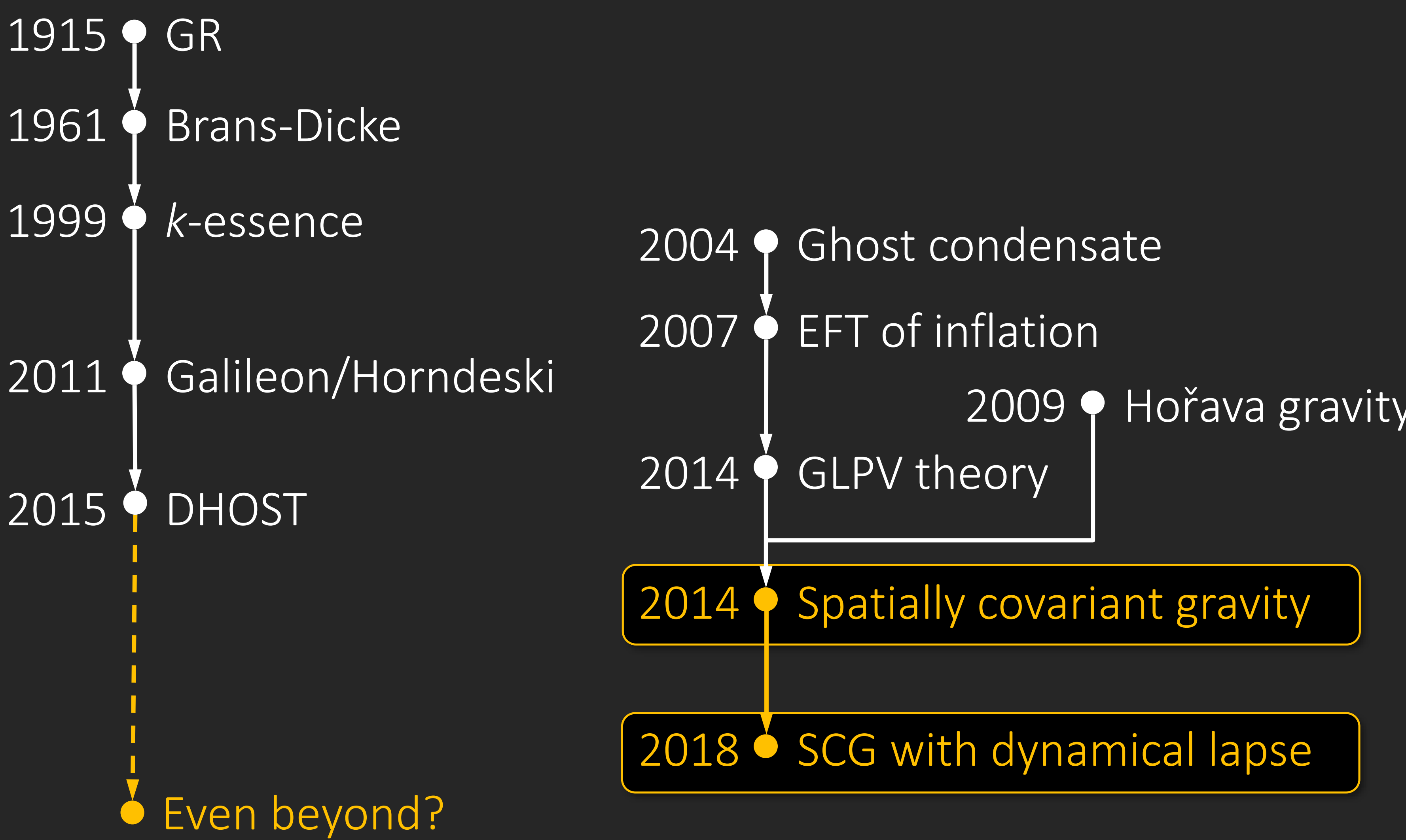

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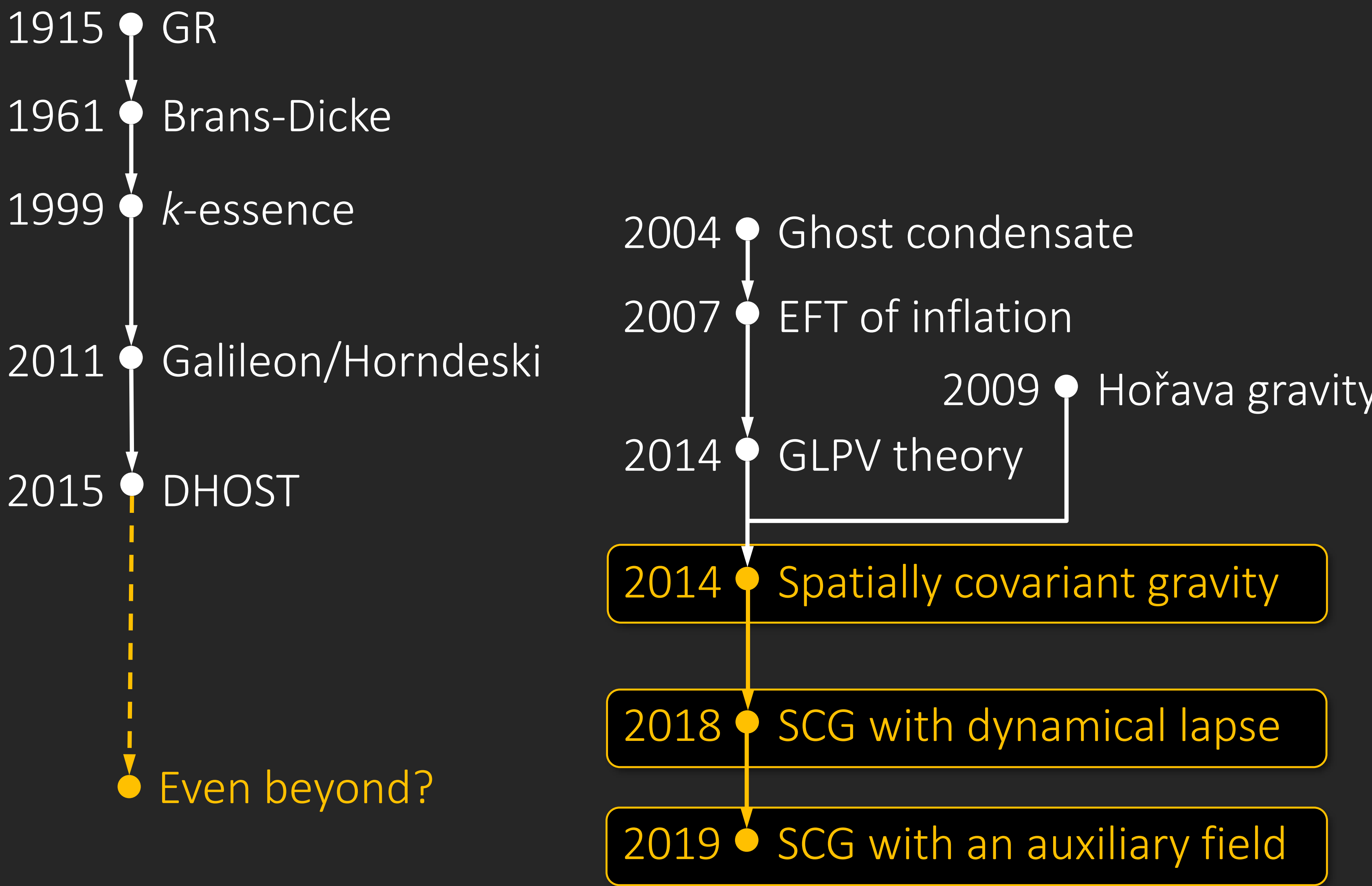

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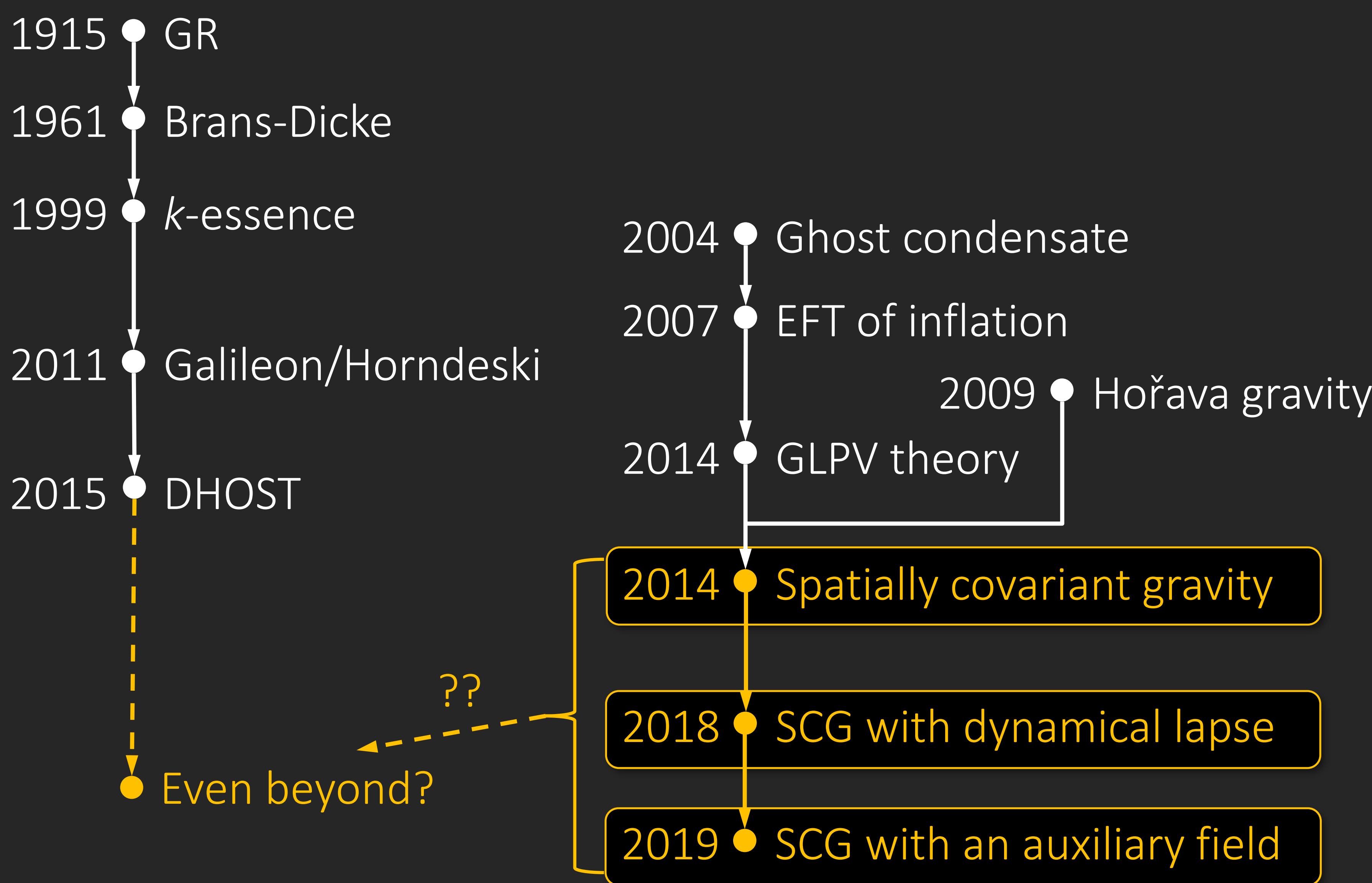

Thank you for your attention!